Exercises

1. Plot the relative populations of the 2nd and ground levels of neutral hydrogen for the temperatures of 4,000 to 20,000.

2. What fraction of the H, He, Mg, Si, Fe, Na, K and Ca are neutral and in the singly ionized condition in the atmospheres of stars of (T = 5,700 and P_e = 30 dynes cm^{-2} ; T = 10,000 and P_e = 300 dynes cm^{-2}) ?

3. The strong Mg II doublet at λ 4481 arises from transitions from the $3^2D_{3/2,5/2}$ term to the 4^2F term. The excitation potential of the lower term of the transition is 8.83 ev, and the ground term of Mg II is $3^2S_{1/2}$. Calculate, for P_e = 100 dynes cm^{-2} and T = 7,200, the fraction of Mg atoms capable of absorbing λ 4481.

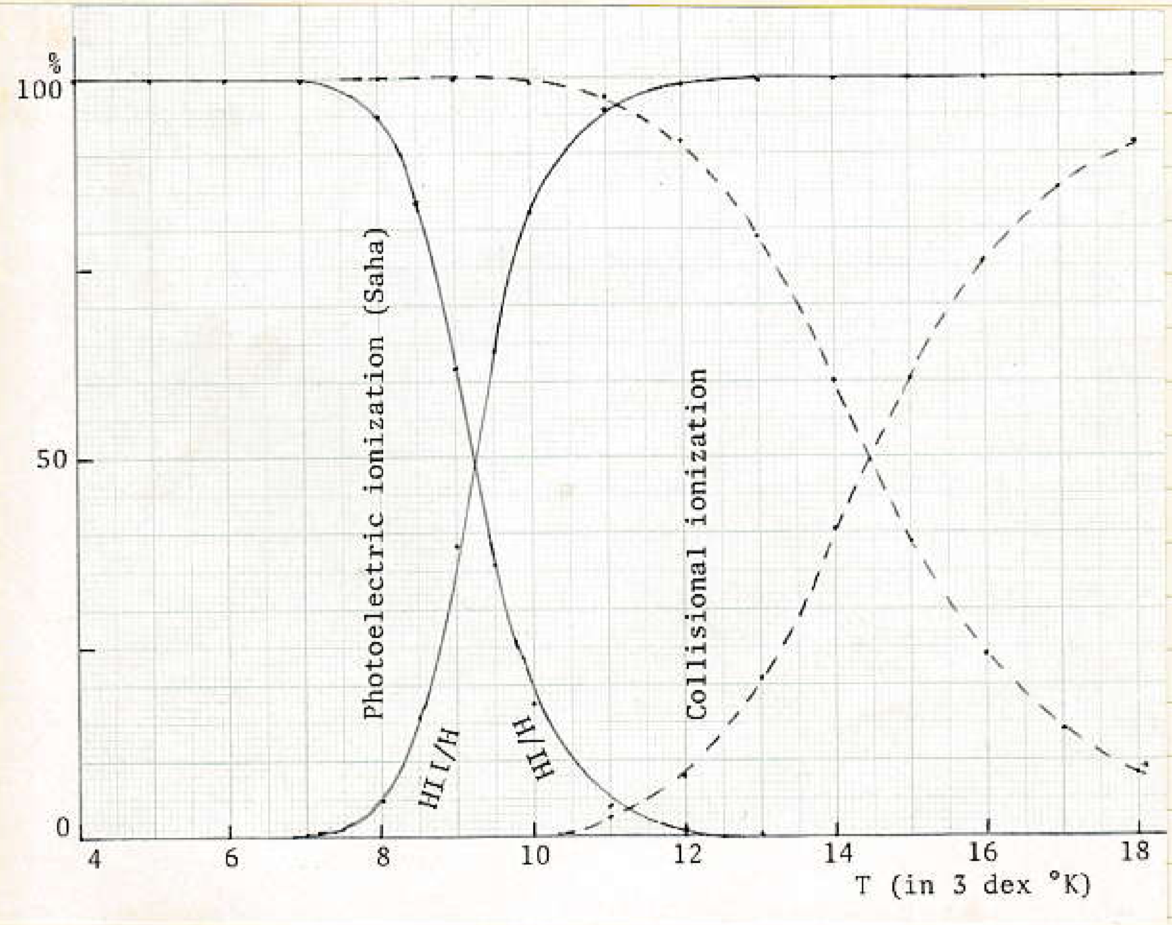
4,000 ~ 30,000
1,000° 간격.

Example (Saha)

Calculate the relative populations of the neutral and ionized hydrogen atoms in the stellar atmospheres for $P_e = 100$ dyn/cm^2, $T_e = 4000$°K to 18000°K with 1000°K-interval.

For hydrogen atom:

$\chi_1 = 13.598$ ev, $g_2 = 1$ and $g_1 = 2$. $P_e = 100$.



T	log II/I	II/I	I/T	II/T
4000	-10.606	2.477-11	1.000	2.477-11
5000	-6.937	1.156-7	1.000	1.156-7
6000	-4.454	3.516-5	1.000	3.515-5
7000	-2.655	2.213-3	9.978-1	2.208-3
8000	-1.286	5.176-2	9.508-1	4.921-2
9000	-0.206	6.223-1	6.164-1	3.836-1
10000	0.669	4.667	1.765-1	8.235-1
11000	1.396	2.489+1	3.863-2	9.614-1
12000	2.010	1.023+2	9.678-3	9.903-1
13000	2.536	3.436+2	2.902-3	9.971-1
14000	2.993	9.840+2	1.015-3	9.990-1
15000	3.394	2.477+3	4.035-4	9.996-1
16000	3.750	5.623+3	1.778-4	9.998-1
17000	4.068	1.169+4	8.550-5	9.999-1
18000	4.354	2.259+4	4.426-5	1.000

Collisional Ionization

Te	I/T	II/T
7000	1.000	4.324-6
8000	1.000	8.271-5
9000	9.992-1	8.318-4
10000	9.947-1	5.312-3
11000	9.760-1	2.402-2
12000	9.184-1	8.164-2
13000	7.907-1	2.093-1
14000	5.958-1	4.042-1
15000	3.935-1	6.065-1
16000	2.397-1	7.603-1
17000	1.424-1	8.576-1
18000	8.564-2	9.144-1
19000	5.297-2	9.470-1
20000	3.390-2	9.661-1

Table 3-5 Ionization of Hydrogen (P_e = 10 dyne cm^{-2}).

T	$\frac{N_{II}}{N_I}$	$\frac{N_I}{N_I + N_{II}}$	$\frac{N_{II}}{N_I + N_{II}}$
4,000°K	2.46×10^{-10}	1.000	0.246×10^{-9}
6,000	3.50×10^{-4}	1.000	0.350×10^{-3}
8,000	5.15×10^{-1}	0.660	0.340
10,000	$4.66 \times 10^{+1}$	0.0210	0.979
12,000	$1.02 \times 10^{+3}$	0.000978	0.999
14,000	$9.82 \times 10^{+3}$	0.000102	1.000
16,000	$5.61 \times 10^{+4}$	0.178×10^{-4}	1.000
18,000	$2.25 \times 10^{+5}$	0.444×10^{-5}	1.000
20,000	$7.05 \times 10^{+5}$	0.142×10^{-5}	1.000

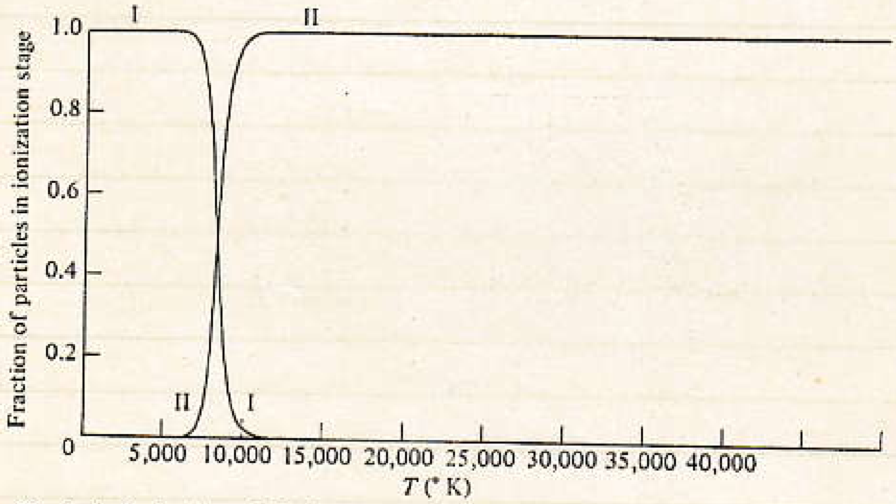
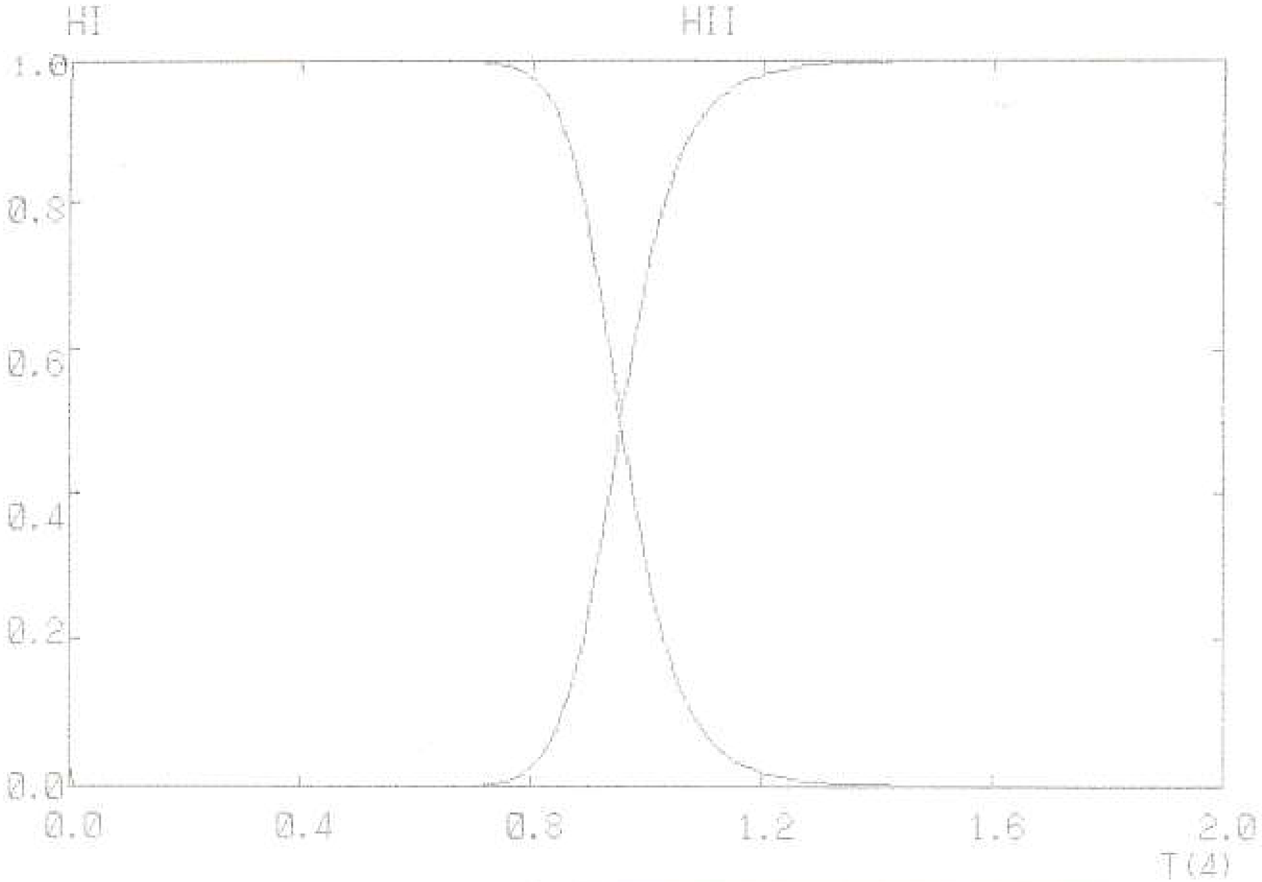


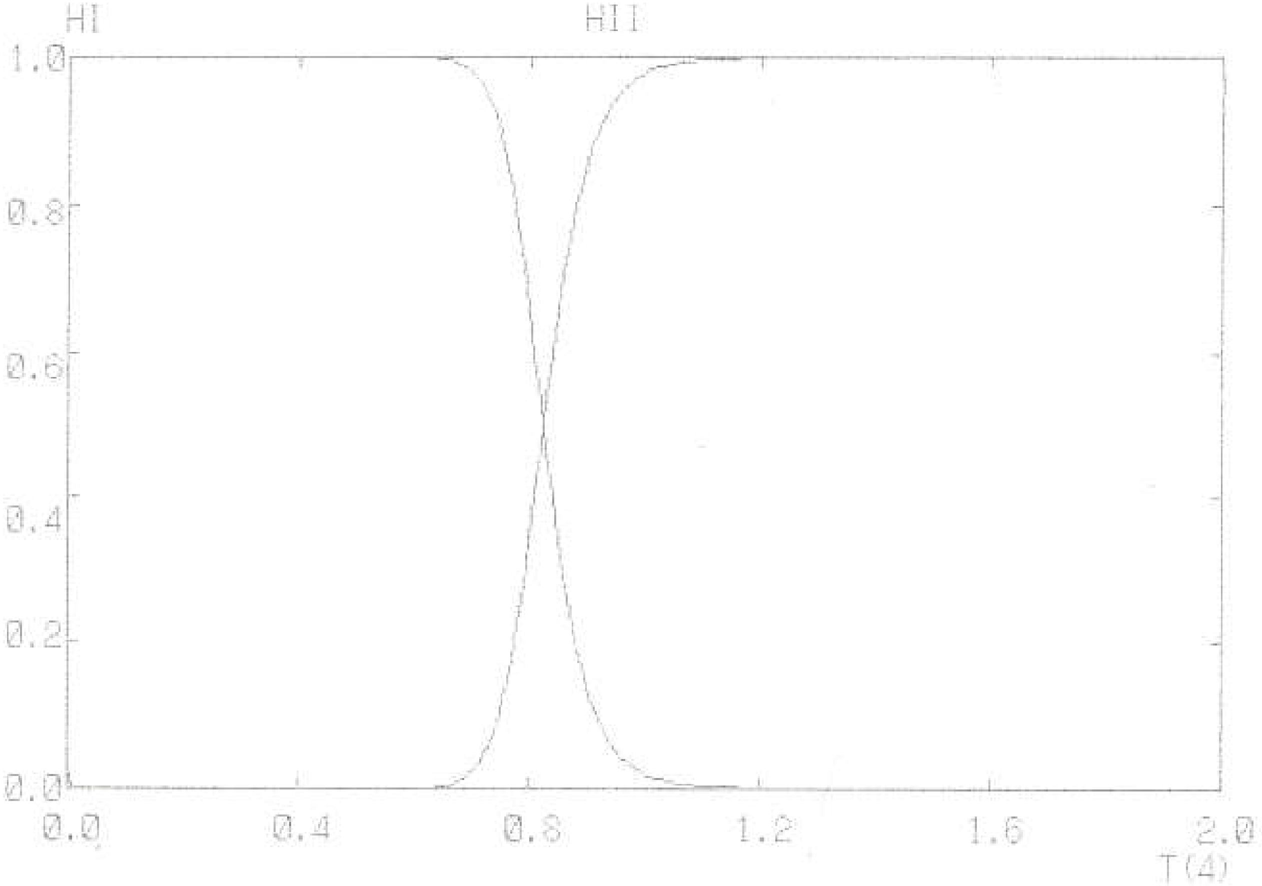
Fig. 3-4 Ionization of Hydrogen. An electron pressure P_e = 10 dynes cm^{-2} is assumed. For this electron pressure, hydrogen is almost completely neutral at temperatures below 6,000°K and almost completely ionized above 11,000°K.

SAHA IONIZATION OF H ATOM
SAHA.AT. VOL.30M. (KOYAMA: 88.05.19)

TEMPERATURE T(4)= 0 TO 2 . ELECTRON PRESSURE Pe= 200 dyne/cm^2.



TEMPERATURE T(4)= 0 TO 2 . ELECTRON PRESSURE Pe= 10 dyne/cm^2.



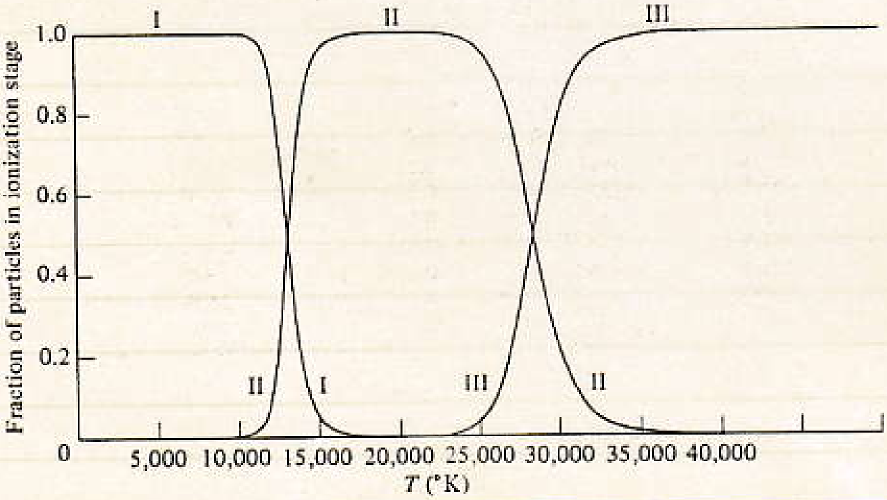
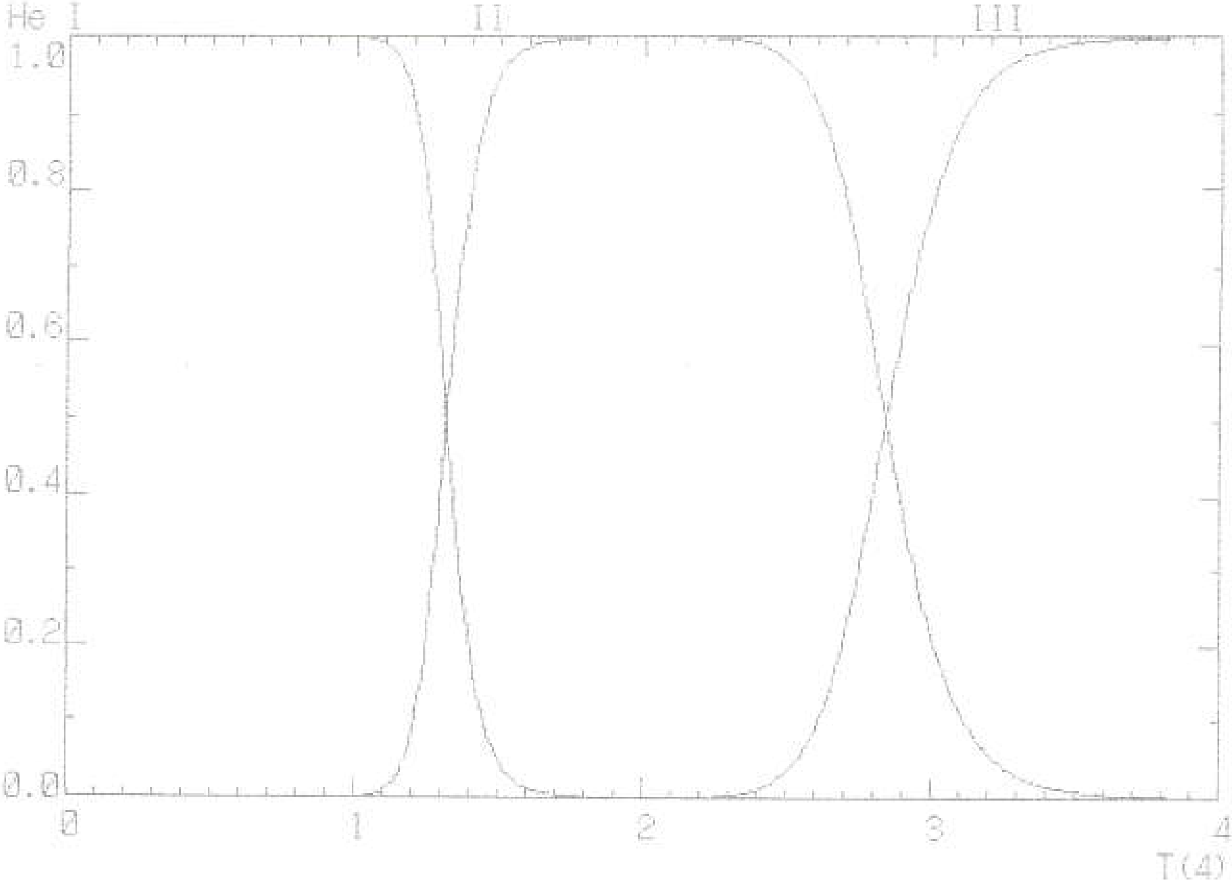


Fig. 3-5 Ionization of Helium. An electron pressure $P_e = 10$ dynes cm^{-2} is assumed. For this electron pressure, helium is almost completely neutral at temperatures below 10,000°K, once ionized in the neighborhood of 20,000°K, and almost entirely twice ionized above 37,000°K.

SAHA IONIZATION OF HELIUM ATOM

SAHA.A1. VOL.30M. (KOYAMA: 88.05.21)

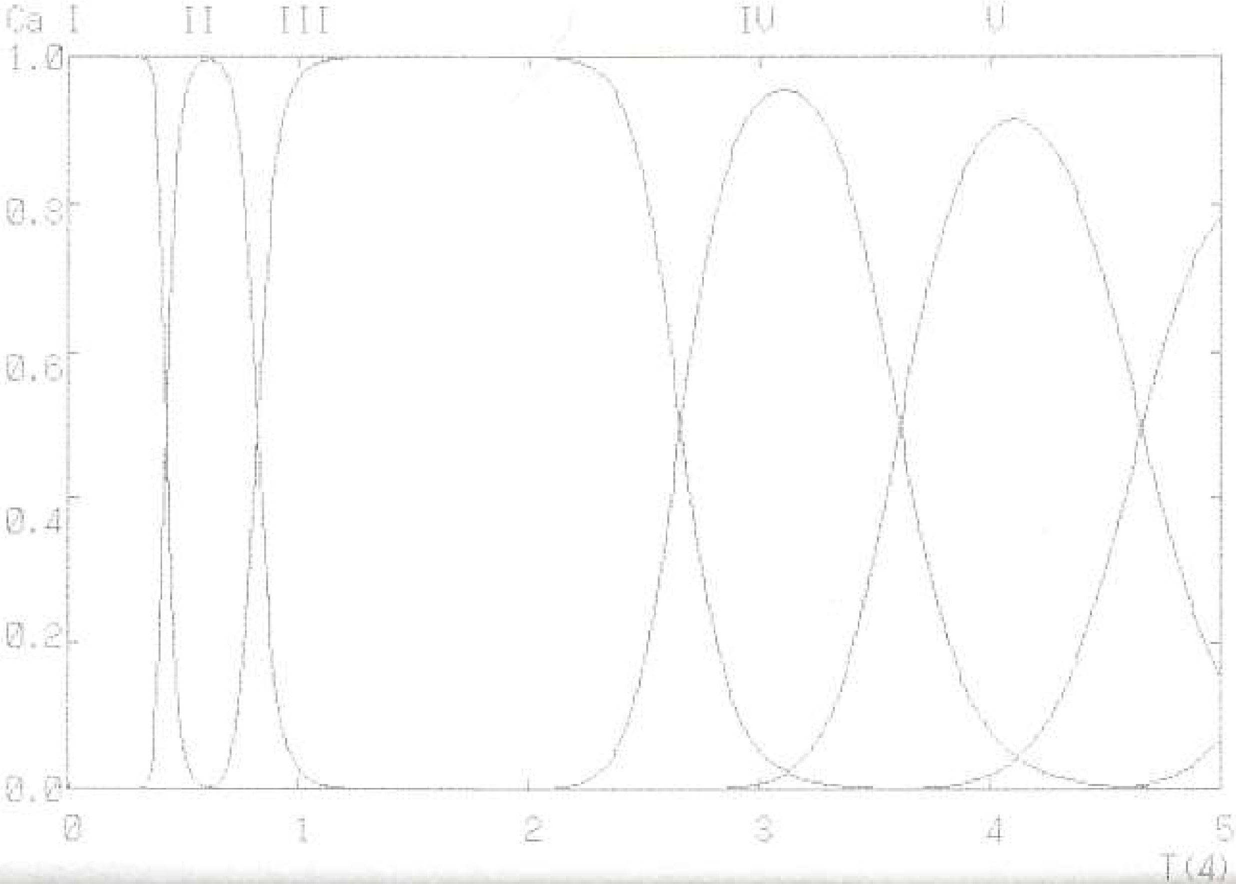
TEMPERATURE T(4)= 0 TO 4 . ELECTRON PRESSURE Pe= 10 dyne/cm^2.



SAHA IONIZATION OF CALCIUM ATOM

SAHA.A1. VOL.30M. (KOYAMA: 88.05.21)

TEMPERATURE T(4)= 0 TO 5 . ELECTRON PRESSURE Pe= 100 dyne/cm^2.

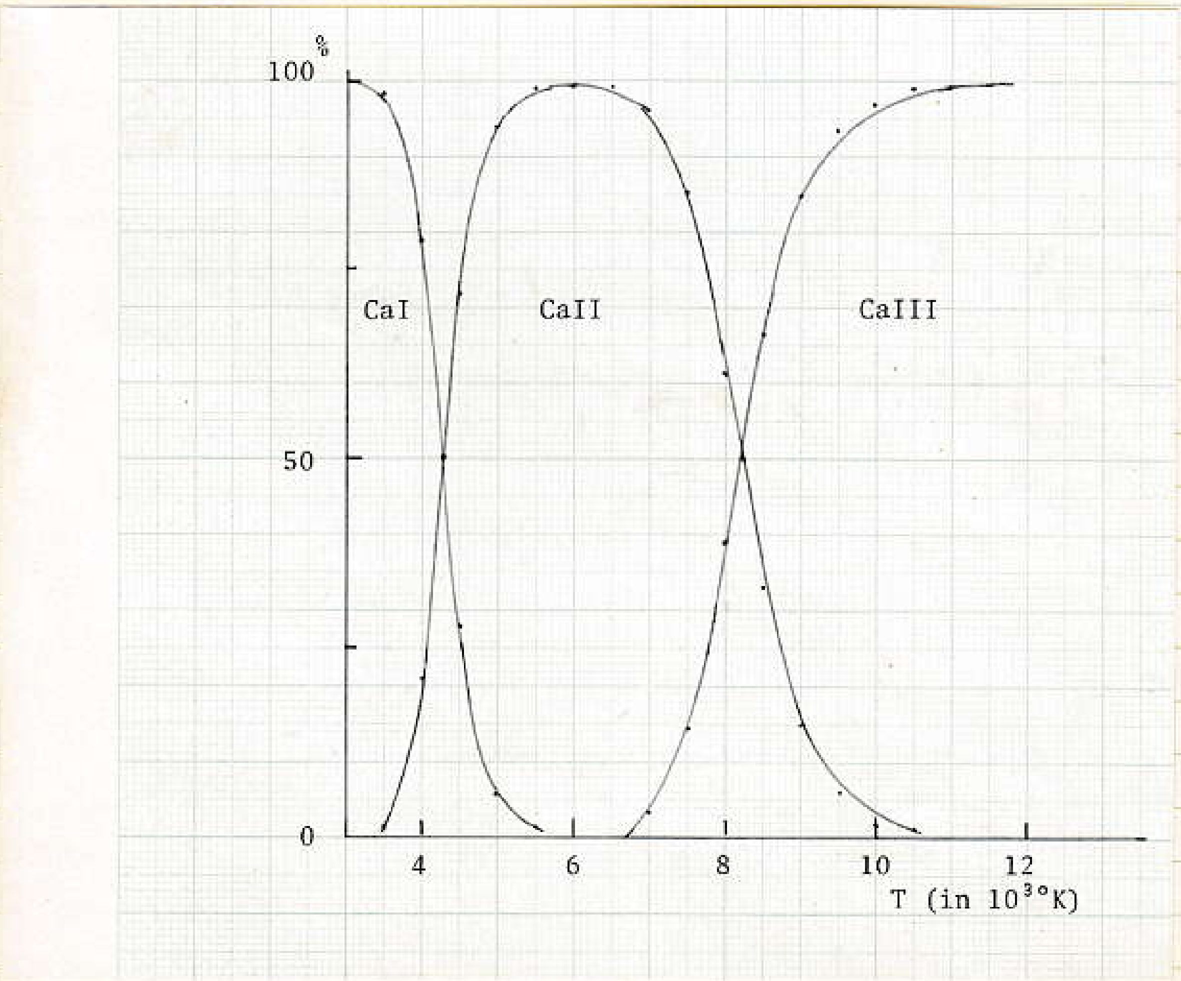


Example (Saha)

In the spectra of cool stars, CaI resonance line 4227 Å is most conspicuous, while in the hot stars, it is replaced by the strong CaII resonance lines 3968 Å (H) and 3934 Å (K), which, in turn, disappear in the hottest stars. To see these process, calculate the degree of ionization of the calcium atoms, for P_e = 100 dyn/cm^2 and T_e = 3,000°K to 11,000°K with 500°K-interval.

From A.Q.3. we get:

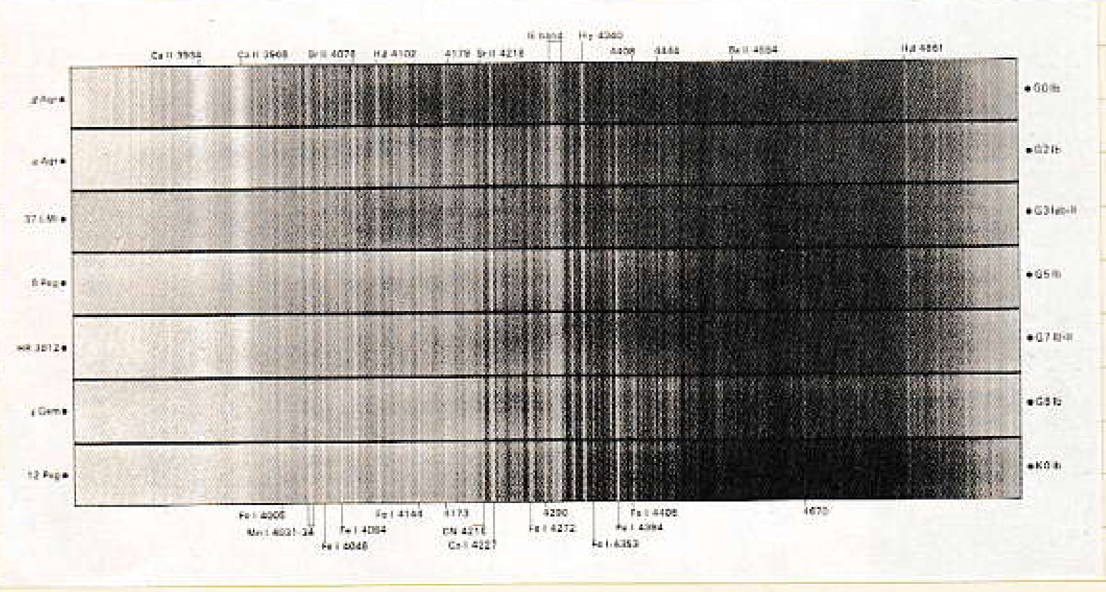
Ca II /Ca I :	χ = 6.113 ev	U_{r+1} = 2	u_r = 1
Ca III/Ca II :	11.871	1	2
Ca IV /Ca III	50.91	6	1



No.

T	II/I	III/II	I/T	II/T	III/T
3000	3.532-4	1.871-14	9.996-1	3.531-4	6.605-18
3500	1.521-2	1.945-11	9.850-1	1.498-2	2.914-13
4000	2.679-1	3.715-9	7.887-1	2.113-1	7.851-10
4500	2.576	2.291-7	2.796-1	7.204-1	1.650-7
5000	1.622+1	6.368-6	5.808-2	9.419-1	5.998-6
5500	7.482+1	9.908-5	1.319-2	9.867-1	9.777-5
6000	2.723+2	9.931-4	3.656-3	9.954-1	9.885-4
6500	8.260+2	7.079-3	1.201-3	9.918-1	7.021-3
7000	2.168+3	3.873-2	4.439-4	9.623-1	3.727-2
7500	5.070+3	1.710-1	1.684-4	8.538-1	1.460-1
8000	1.074+4	6.339-1	5.698-5	6.120-1	3.879-1
8500	2.109+4	2.032	1.564-5	3.298-1	6.702-1
9000	3.864+4	5.768	3.824-6	1.478-1	8.522-1
9500	6.699+4	1.476+1	9.474-7	6.346-2	9.365-1
10000	1.107+5	3.467+1	2.533-7	2.803-2	9.720-1
10500	1.754+5	7.551+1	7.452-8	1.307-2	9.869-1
11000	2.673+5	1.538+2	2.416-8	6.459-3	9.935-1

このgraphをもとにして、Antares、Sun、Siriusのspectra中のCa線について考える。三者の星の有効温度は、夫々3,500°K、5800°K、10,400°Kである。温度の低いAntaresでは、Caの電離は、まだ殆ど始まっておらず、CaⅠの状態にある。Sunでは、電離が進んでおり、殆どCaⅡの状態であるが、まだCaⅠも相当残っている。高温なSiriusでは、CaⅡ、CaⅢの状態にあり、CaⅠは実質上無くなっている。CaⅠ 4227；CaⅡ 3968(H)、CaⅡ 3934(K)の吸収線を注意すると、AntaresではCaⅠだけ、Sunでは、CaⅠ、CaⅡの両方、Siriusでは、CaⅡだけが現れている。



exercise

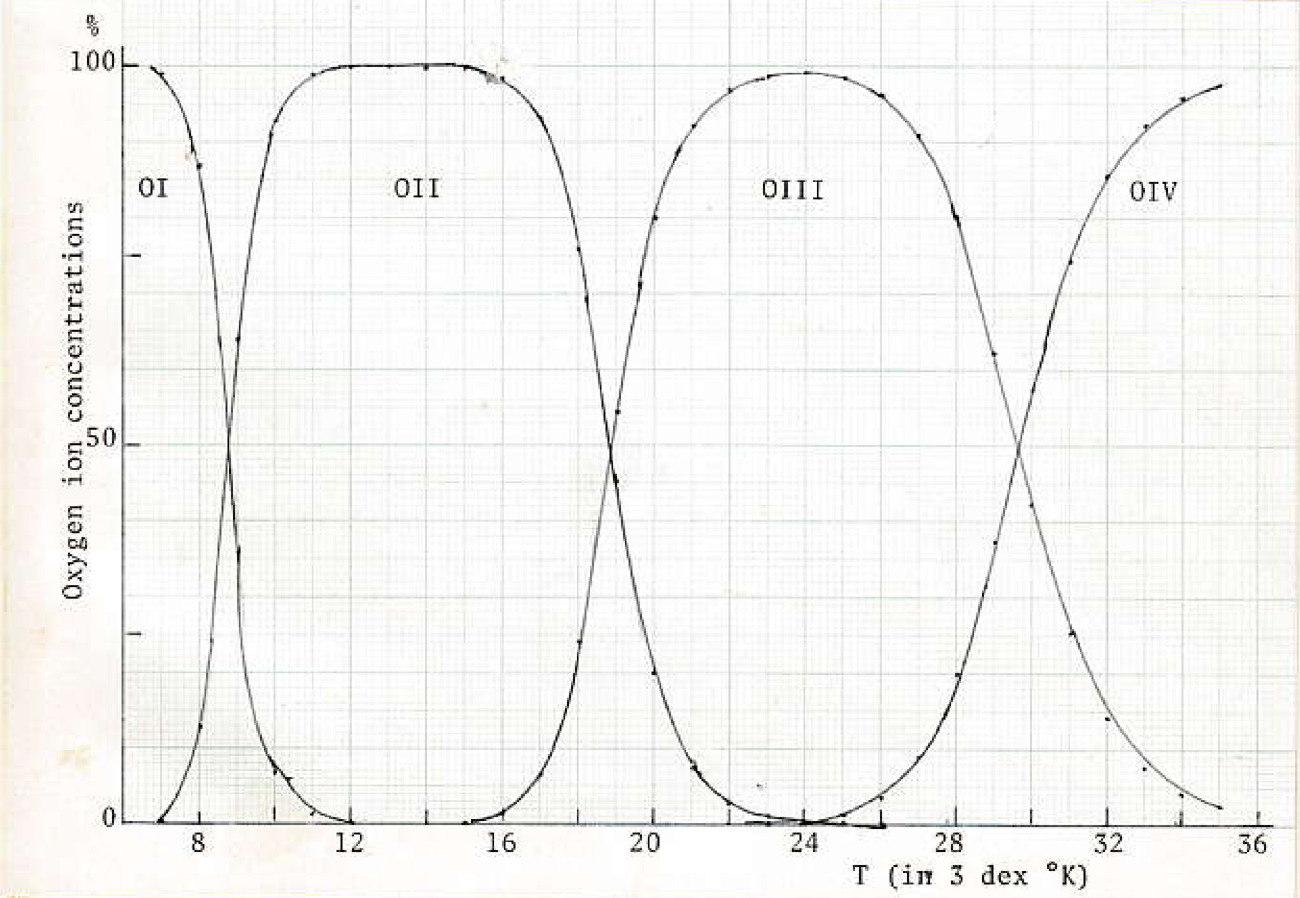
No.

Example (Saha)

What fraction (%) of oxygen atoms will be in each states of ionization in the atmospheres of stars with T = 7000°K to 35000°K and with $p_e = 30$ dyn/cm^2.

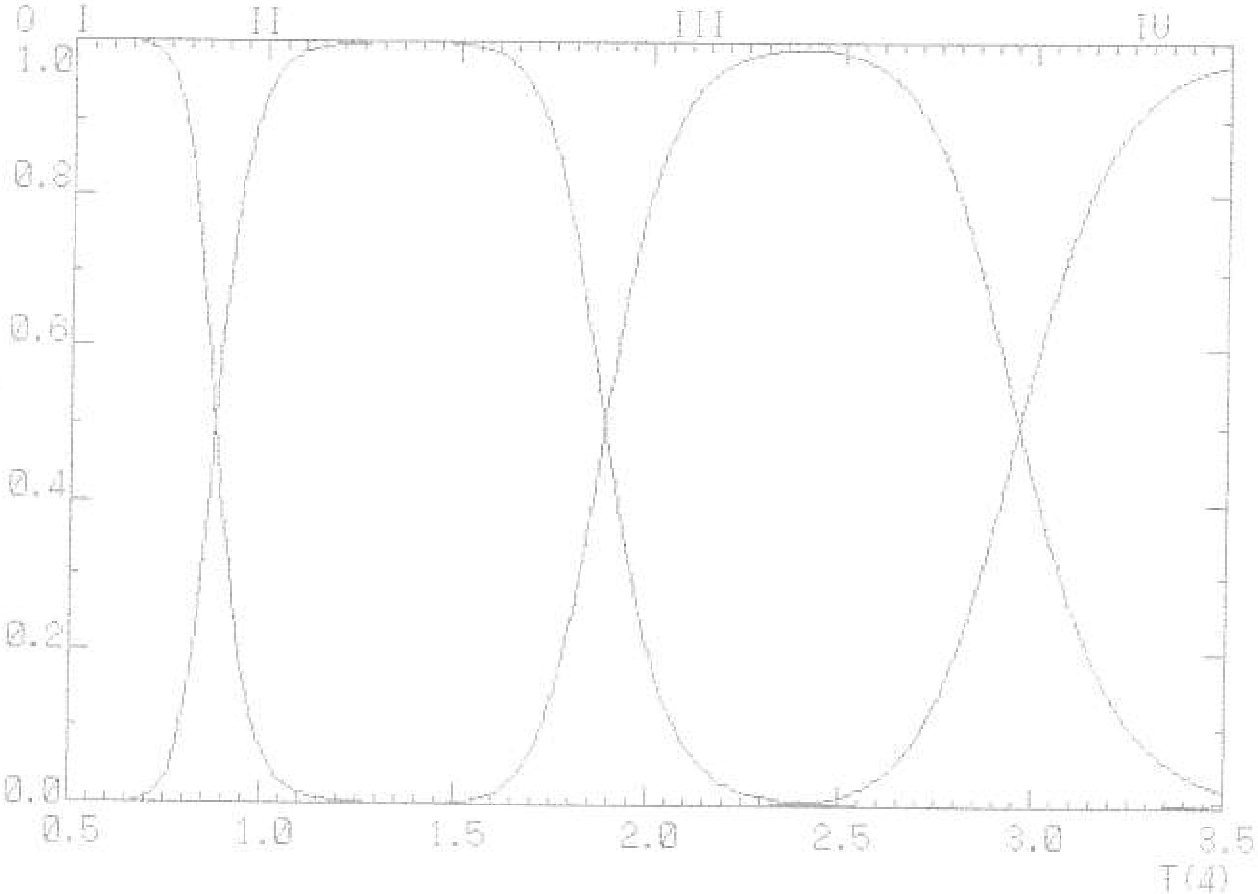
From AQ3, we get:

	X_r	g_{r+1}	g_r
OII/OI	13.618 ev	4	9
OIII/OII	35.117	9	4
OIV/OIII	54.934	6	9



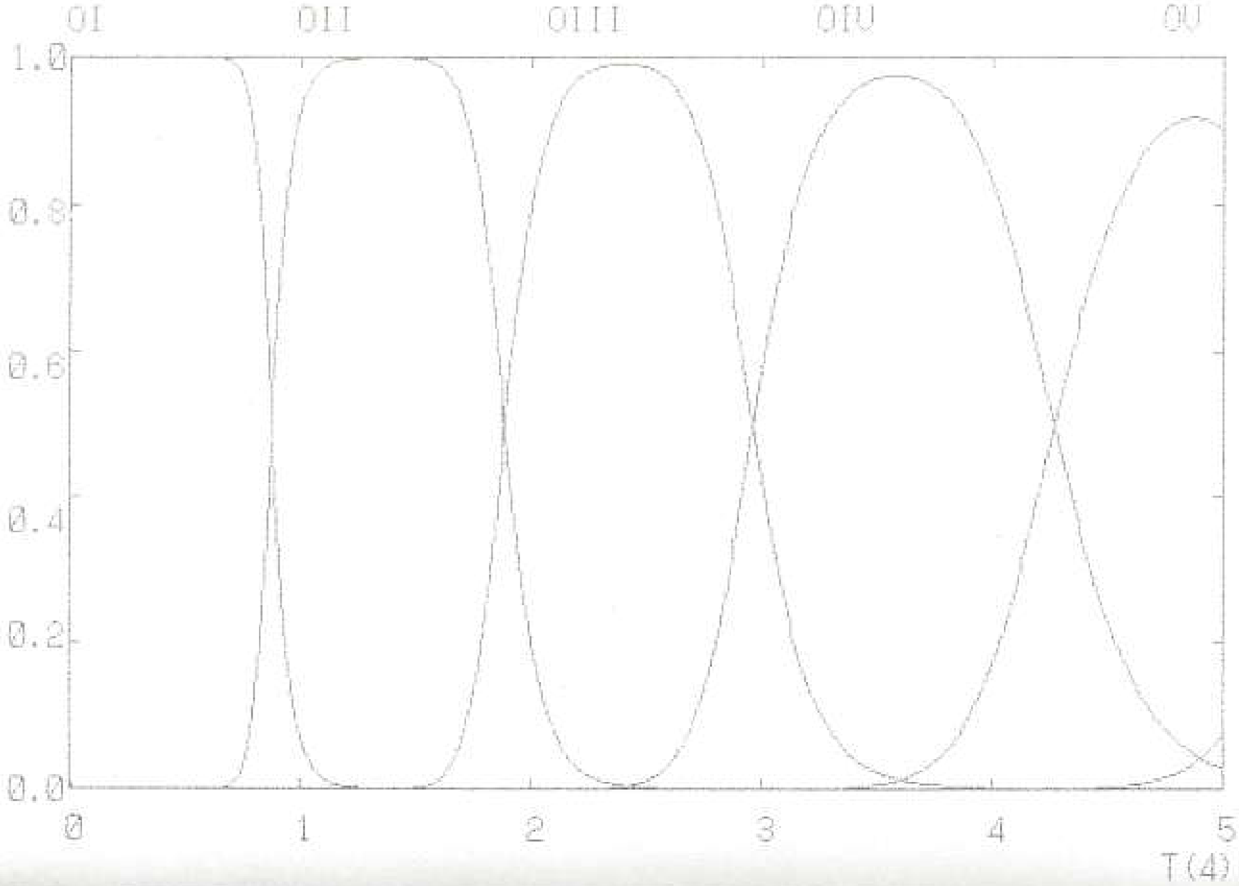
No.

T	log II/I	logIII/II	logIV/III	I/T	II/T	III/T	IV/T
7000	-2.1976	-16.9725	-31.7690	9.937-1	6.305-3	6.717-20	1.143-51
8000	-0.8270	-13.6670	-26.6800	8.704-1	1.296-1	2.791-15	5.830-42
9000	0.2541	-11.0810	-22.7067	3.578-1	6.422-1	5.330-12	1.047-34
10000	1.1311	-9.0000	-19.5161	6.885-2	9.311-1	9.311-10	2.837-29
11000	1.8586	-7.2876	-16.8956	1.366-2	9.863-1	5.087-8	6.469-25
12000	2.4730	-5.8523	-14.7037	3.354-3	9.966-1	1.400-6	2.770-21
13000	2.9999	-4.6308	-12.8420	9.992-4	9.990-1	2.338-5	3.363-18
14000	3.4574	-3.5779	-11.2403	3.486-4	9.994-1	2.641-4	1.519-15
15000	3.8592	-2.6602	-9.8469	1.380-4	9.977-1	2.182-3	3.104-13
16000	4.2152	-1.8526	-8.6232	6.008-5	9.861-1	1.385-2	3.297-11
17000	4.5334	-1.1361	-7.5395	2.729-5	9.319-1	6.812-2	1.967-9
18000	4.8197	-0.4957	-6.5727	1.148-5	7.579-1	2.421-1	6.475-8
19000	5.0791	0.0805	-5.7044	3.782-6	4.538-1	5.462-1	1.079-6
20000	5.3154	0.6020	-4.9201	9.675-7	2.000-1	8.000-1	9.616-6
21000	5.5318	1.0764	-4.2080	2.274-7	7.737-2	9.226-1	5.715-5
22000	5.7309	1.5100	-3.5582	5.569-8	2.997-2	9.698-1	2.682-4
23000	5.9148	1.9080	-2.9627	1.484-8	1.220-2	9.867-1	1.075-3
24000	6.0853	2.2749	-2.4150	4.324-9	5.262-3	9.909-1	3.811-3
25000	6.2440	2.6142	-1.9092	1.366-9	2.396-3	9.855-1	1.215-2
26000	6.3922	2.9290	-1.4407	4.601-10	1.135-3	9.639-1	3.494-2
27000	6.5309	3.2221	-1.0053	1.606-10	5.454-4	9.096-1	8.986-2
28000	6.6612	3.4957	-0.5996	5.567-11	2.551-4	7.989-1	2.009-1
29000	6.7838	3.7518	-0.2205	1.819-11	1.105-4	6.242-1	3.757-1
30000	6.8995	3.9920	0.1345	5.433-12	4.310-5	4.232-1	5.768-1
31000	7.0089	4.2179	0.4678	1.507-12	1.538-5	2.540-1	7.459-1
32000	7.1126	4.4308	0.7814	4.062-13	5.264-6	1.419-1	8.581-1
33000	7.2110	4.6318	1.0770	1.110-13	1.804-6	7.728-2	9.227-1
34000	7.3046	4.8220	1.3562	3.151-14	6.355-7	4.218-2	9.578-1
35000	7.3937	5.0022	1.6203	9.408-15	2.329-7	2.341-2	9.766-1



SAHA IONIZATION OF OXYGEN ATOM
SAHA.A2. VOL.30M. (KOYAMA: 88.05.23)

TEMPERATURE T(4)= .5 TO 3.5 . ELECTRON PRESSURE Pe= 30 dyne/cm^2.



SAHA IONIZATION OF O ATOM
SAHA.A1. VOL.30M. (KOYAMA: 88.05.19)

TEMPERATURE T(4)= 0 TO 5 . ELECTRON PRESSURE Pe= 30 dyne/cm^2.

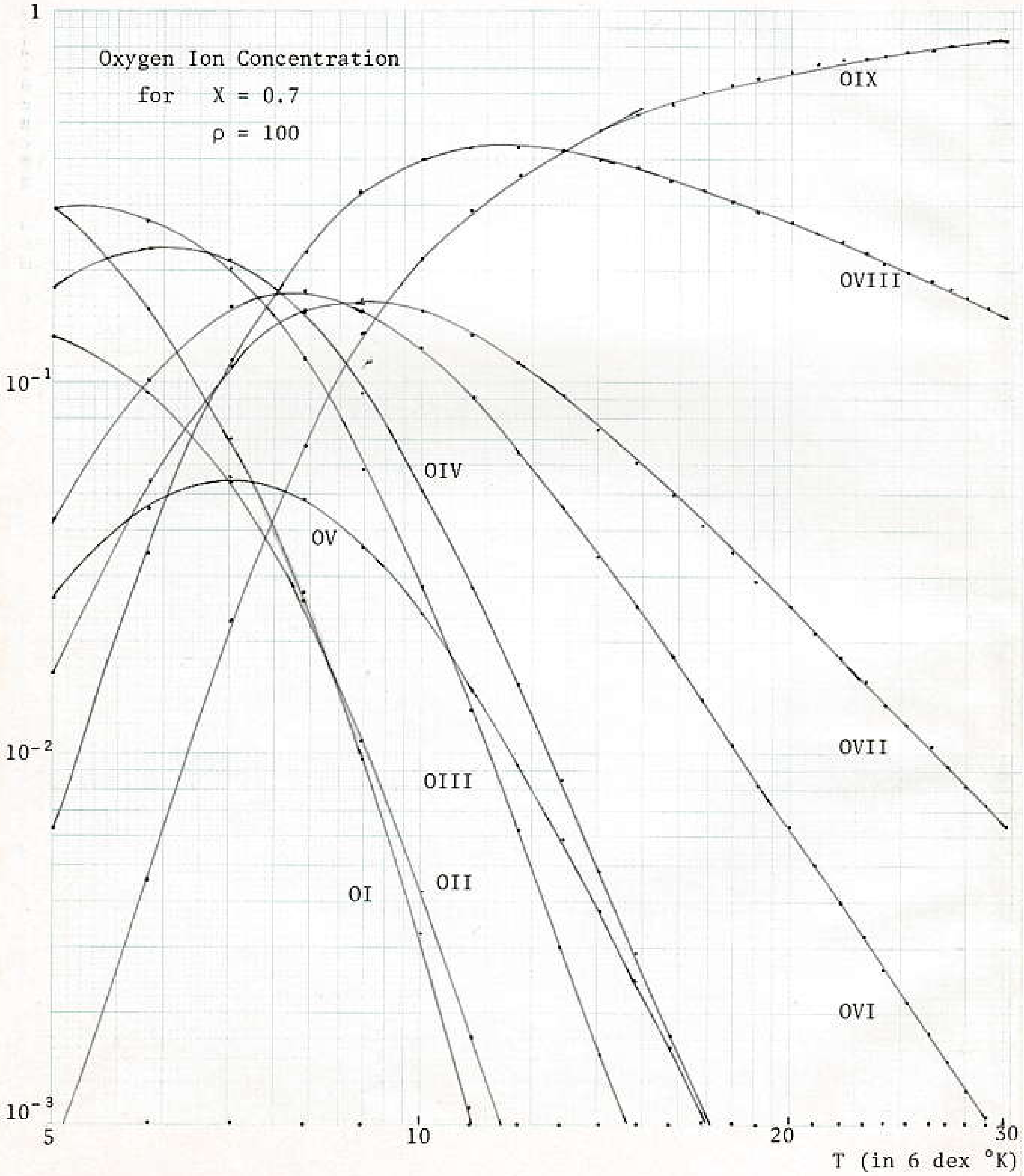
Oxygen ion concentration

T in 6 dex°K	I/T	I/T	II/T	III/T	IV/T	V/T	VI/T	VII/T	VIII/T	IX/T
5	2.960 907 017-1	2.961-1	1.345-1	2.941-1	1.820-1	2.637-2	4.328-2	1.656-2	6.281-3	4.383-4
6	1.579 650 900-1	1.580-1	9.479-2	2.762-1	2.295-1	4.566-2	1.016-1	5.390-2	3.576-2	4.595-3
7	7.144 740 551-2	7.145-2	5.424-2	2.011-1	2.138-1	5.476-2	1.584-1	1.100-1	1.129-1	2.325-2
8	2.762 615 348-2	2.763-2	2.569-2	1.173-1	1.541-1	4.899-2	1.773-1	1.548-1	2.261-1	6.817-2
9	9.680 623 548-3	9.681-3	1.077-2	5.897-2	9.326-2	3.583-2	1.575-1	1.679-1	3.296-1	1.365-1
10	3.302 480 041-3	3.302-3	4.310-3	2.777-2	5.180-2	2.354-2	1.230-1	1.563-1	3.954-1	2.145-1
11	1.152 394 212-3	1.152-3	1.737-3	1.296-2	2.806-2	1.483-2	9.051-2	1.346-1	4.247-1	2.914-1
12	4.210 532 090-4	4.211-4	7.242-4	6.175-3	1.531-2	9.280-3	6.518-2	1.118-1	4.290-1	3.621-1
13	1.624 636 496-4	1.625-4	3.154-4	3.041-3	8.532-3	5.868-3	4.687-2	9.159-2	4.186-1	4.251-1
14	6.625 550 399-5	6.626-5	1.439-4	1.553-3	4.888-3	3.775-3	3.394-2	7.479-2	4.004-1	4.804-1
15	2.847 055 339-5	2.847-5	6.861-5	8.232-4	2.882-3	2.479-3	2.488-2	6.125-2	3.788-1	5.288-1
16	1.283 750 115-5	1.284-5	3.410-5	4.515-4	1.746-3	1.661-3	1.846-2	5.042-2	3.560-1	5.712-1
17	6.054 542 666-6	6.055-6	1.763-5	2.560-4	1.087-3	1.136-3	1.389-2	4.179-2	3.336-1	6.082-1
18	2.973 941 505-6	2.974-6	9.437-6	1.495-4	6.928-4	7.913-4	1.059-2	3.489-2	3.121-1	6.408-1
19	1.514 475 101-6	1.514-6	5.214-6	8.969-5	4.517-4	5.610-4	8.175-3	2.935-2	2.919-1	6.695-1
20	7.981 541 565-7	7.982-7	2.969-6	5.522-5	3.008-4	4.044-4	6.387-3	2.486-2	2.731-1	6.948-1
21	4.334 629 908-7	4.335-7	1.736-6	3.477-5	2.041-4	2.958-4	5.043-3	2.121-2	2.558-1	7.174-1
22	2.421 717 063-7	2.422-7	1.040-6	2.236-5	1.409-4	2.195-4	4.024-3	1.821-2	2.400-1	7.374-1
23	1.389 147 767-7	1.389-7	6.379-7	1.467-5	9.898-5	1.650-4	3.242-3	1.573-2	2.254-1	7.554-1
24	8.154 739 304-8	8.155-8	3.993-7	9.795-6	7.052-5	1.255-4	2.635-3	1.367-2	2.120-1	7.715-1
25	4.893 188 149-8	4.893-8	2.548-7	6.650-6	5.094-5	9.658-5	2.160-3	1.194-2	1.998-1	7.859-1
26	2.996 194 320-8	2.996-8	1.655-7	4.583-6	3.728-5	7.505-5	1.784-3	1.049-2	1.886-1	7.990-1
27	1.869 336 413-8	1.869-8	1.093-7	3.205-6	2.761-5	5.891-5	1.485-3	9.255-3	1.782-1	8.109-1
28	1.186 134 069-8	1.186-8	7.325-8	2.270-6	2.067-5	4.664-5	1.243-3	8.202-3	1.687-1	8.217-1
29	7.654 148 743-9	7.654-9	4.983-8	1.628-6	1.564-5	3.723-5	1.048-3	7.303-3	1.600-1	8.316-1
30	5.012 890 633-9	5.013-9	3.435-8	1.181-6	1.195-5	2.996-5	8.884-4	6.525-3	1.519-1	8.406-1

Hydrogen abundance X = 0.7

Density $\rho = 100$

No.

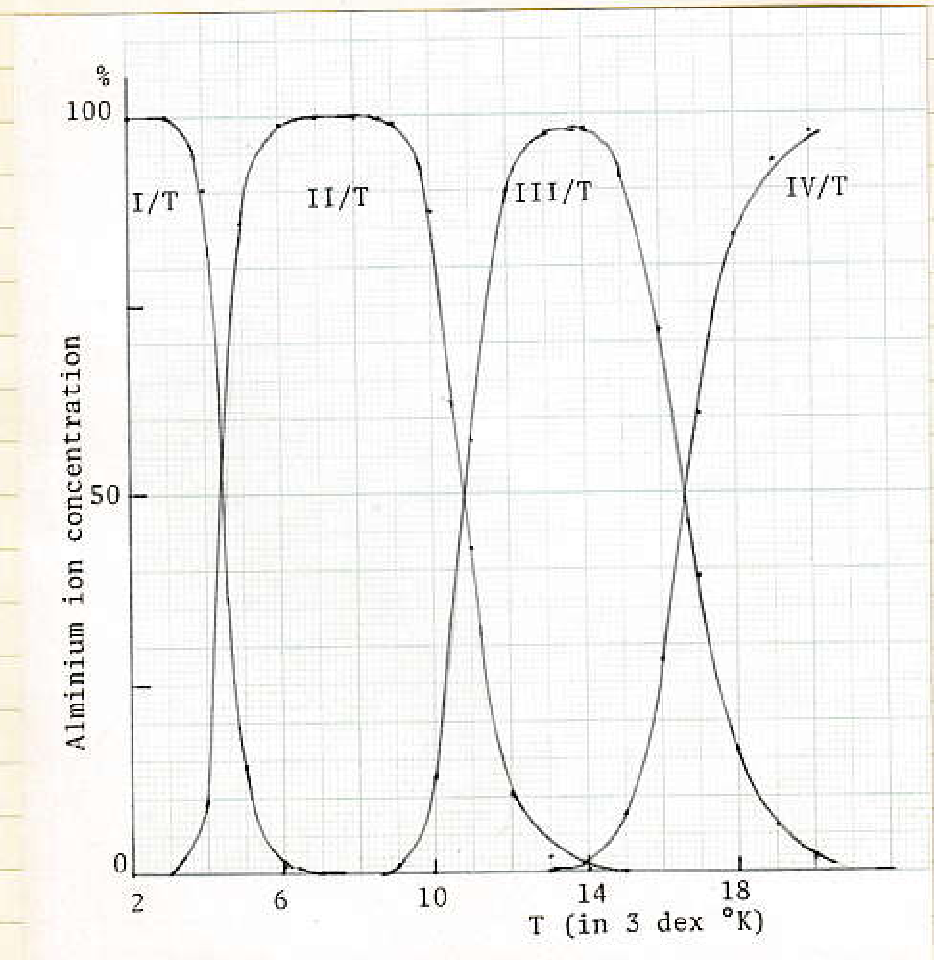

Oxygen Ion Concentration
for X = 0.7
ρ = 100
1
10⁻¹
10⁻²
10⁻³
5
10
20
30
T (in 6 dex °K)
OIX
OVIII
OVII
OVI
OV
OIV
OIII
OII
OI

Example (Saha)

What fraction (%) of alminium atoms will be in each states of ionization in the atmosphere of stars with T = 2000°K to 20000°K and with $p_e = 30$ dyn/cm^2.

From AQ3, we get:

	X_r	g_{r+1}	g_r
Al II/Al I	5.986	1	6
Al III/Al II	18.826	2	1
Al IV/Al III	28.448	1	2



T	I/T	II/T	III/T	IV/T
2000	1.000	5.451-10	1.568-50	
3000	9.998-1	1.603-4	8.278-29	7.309-70
4000	9.029-1	9.708-2	8.305-18	1.336-46
5000	1.417-1	8.583-1	7.116-12	2.950-33
6000	1,023-2	9.898-1	1.883-8	7.407-25
7000	1.343-3	9.987-1	5.072-6	7.605-19
8000	2.783-4	9.994-1	3.506-4	2.667-14
9000	7.828-5	9.902-1	9.695-3	9.706-11
10000	2.453-5	8.738-1	1.262-1	6.441-8
11000	5.038-6	4.282-1	5.717-1	7.449-6
12000	5.772-7	1.032-1	8.966-1	1.771-4
13000	6.645-8	2.266-2	9.754-1	1.953-3
14000	9.472-9	5.694-3	9.798-1	1.449-2
15000	1.599-9	1.591-3	9.205-1	7.788-2
16000	2.720-10	4.244-4	7.173-1	2.822-1
17000	3.822-11	8.962-5	3.935-1	6.065-1
18000	4.572-12	1.552-5	1.605-1	8.395-1
19000	5.603-13	2.667-6	5.982-2	9.402-1
20000	7.792-14	5.062-7	2.294-2	9.771-1

Exercise 1.

O^{++} ion の基底状態は $1s^2 2s^2 2p^2\ {}^3P_0$ である。発光電子が 3d 電子となった すべての励起状態の名称を完全な形で書け。又、この励起状態の中で、基底状態への遷移が許されるのは どれか。

Exercise 2.

上で考えた励起状態の一つは、基底状態への遷移に際し 波長 λ 305, 597 3765 Å の極紫外輻射を放出する。温度 $T = 15{,}000^\circ$, $16{,}000^\circ$ ……, $20{,}000^\circ K$. 電子圧 $P_e = 10$ dynes/cm^2 の恒星大気内で この励起状態に励起されている O^{++} ion の数は、O 総数の中 どの位の割合か。

問題の励起状態の電子配置は $1s^2 2s^2 2p\, 3d$ である。この中、1s 殻、2s 殻は、すでに それぞれ 2ケの電子が入って殻は完成され、$L = 0$, $S = 0$, $J = 0$ (${}^1S_0$) であるから、残りの 2p 電子、3d 電子について、ベクトル的に、l が合成されて L を作り、s が合成されて S を作る。すなわち、

$$\mathbb{L} = \vec{1}(p) + \vec{2}(d) = 1\ (P\text{項}),\ 2\ (D\text{項}),\ 3\ (F\text{項});$$

$$\mathbb{S} = \vec{\tfrac{1}{2}} + \vec{\tfrac{1}{2}} = 0\ (\text{従って}\ r=1,\ \text{singlet}),\ 1\ (\text{従って}\ r=3,\ \text{triplet}).$$

従って、先ず項は ${}^1P$, ${}^1D$, ${}^1F$; ${}^3P$, ${}^3D$, ${}^3F$ の 6 種類。

各項について、L, S, を合成して J をもとめる。 $\mathbb{J} = \mathbb{L} + \mathbb{S}$

${}^1P$ ($L=1$, $S=0$) $J = 1$ ${}^1P_1^o$

${}^1D$ ($L=2$, $S=0$) $J = 2$ ${}^1D_2^o$

${}^1F$ ($L=3$, $S=0$) $J = 3$ ${}^1F_3^o$

${}^3P$ ($L=1$, $S=1$) $J = 0, 1, 2$ ${}^3P_0^o$ ${}^3P_1^o$ ${}^3P_2^o$

${}^3D$ ($L=2$, $S=1$) $J = 1, 2, 3$ ${}^3D_1^o$ ${}^3D_2^o$ ${}^3D_3^o$

${}^3F$ ($L=3$, $S=1$) $J = 2, 3, 4$ ${}^3F_2^o$ ${}^3F_3^o$ ${}^3F_4^o$

又、parity $\Sigma l = 1(p) + 2(d) = 3$: odd

之等の中 選択規則によって 基底状態への遷移が許されるのは、

${}^3D_1^o$, ${}^3P_1^o$ の二つである。

AQ3の修正必要

次に．O原子について

$\chi_0 = 13.\overset{618}{614}$ eV．　$g_{1,1} = 4$．　$g_{0,1} = 9$．

$\chi_1 = 35.\overset{117}{108}$ eV．　$g_{2,1} = 9$．　$g_{1,1} = 4$．

$\chi_2 = 54.\overset{934}{886}$ eV．　$g_{3,1} = 6$．　$g_{2,1} = 9$．

をSahaの式に入れて．O総数に対するO^{++}の数．$N_2/\Sigma N_i$ を求める．（例題参照）。

次に．λ 305.597 3965 Å（305.597 3965 × 10^{-8} cm）．の輻射に対する状態間のenergy差．すなわち，問題の励起状態の励起ポテンシャルは，

$$\chi = 1/\lambda \ \mathrm{cm}^{-1} = 327\,227.94 \ \mathrm{cm}^{-1}$$
$$= 40.57 \ \mathrm{eV}.$$

ground termのstatistical weight $g_{2,1} = 9$．
この励起状態（$J = 1$）．のstatistical weight $g_J = 2J+1 = 3$．
これから．Boltzmannの式により．O^{++} ion基底状態の数（～O^{++} ion総数）に対する励起ionの数 $N_{ex.}/N_2$ が得られる。

上に求めた $N_2/\Sigma N_i$ と N_{ex}/N_2 をかければ．$N_{ex}/\Sigma N_i$ が出る。

T.	$N_2/\Sigma N_i$	N_{ex}/N_2.	$N_{ex}/\Sigma N_i$
15,000	$6.56\cdot10^{-3}$	$7.78\cdot10^{-15}$	$5.10\cdot10^{-17}$
16,000	$4.06\cdot10^{-2}$	$5.53\cdot10^{-14}$	$2.25\cdot10^{-15}$
17.000	$1.81\cdot10^{-1}$	$3.13\cdot10^{-13}$	$5.67\cdot10^{-14}$
18,000	$4.91\cdot10^{-1}$	$1.46\cdot10^{-12}$	$7.17\cdot10^{-13}$
19,000	$7.84\cdot10^{-1}$	$5.77\cdot10^{-12}$	$4.52\cdot10^{-12}$
20,000	$9.23\cdot10^{-1}$	$1.99\cdot10^{-11}$	$1.84\cdot10^{-11}$
21,000	$9.73\cdot10^{-1}$	$6.11\cdot10^{-11}$	$5.95\cdot10^{-11}$
22,000	$9.89\cdot10^{-1}$	$1.69\cdot10^{-10}$	$1.67\cdot10^{-10}$
23,000	$9.93\cdot10^{-1}$	$4.30\cdot10^{-10}$	$4.27\cdot10^{-10}$
24,000	$9.87\cdot10^{-1}$	$1.01\cdot10^{-9}$	$9.97\cdot10^{-10}$
25,000	$9.63\cdot10^{-1}$	$2.21\cdot10^{-9}$	$2.13\cdot10^{-9}$.

Exercise

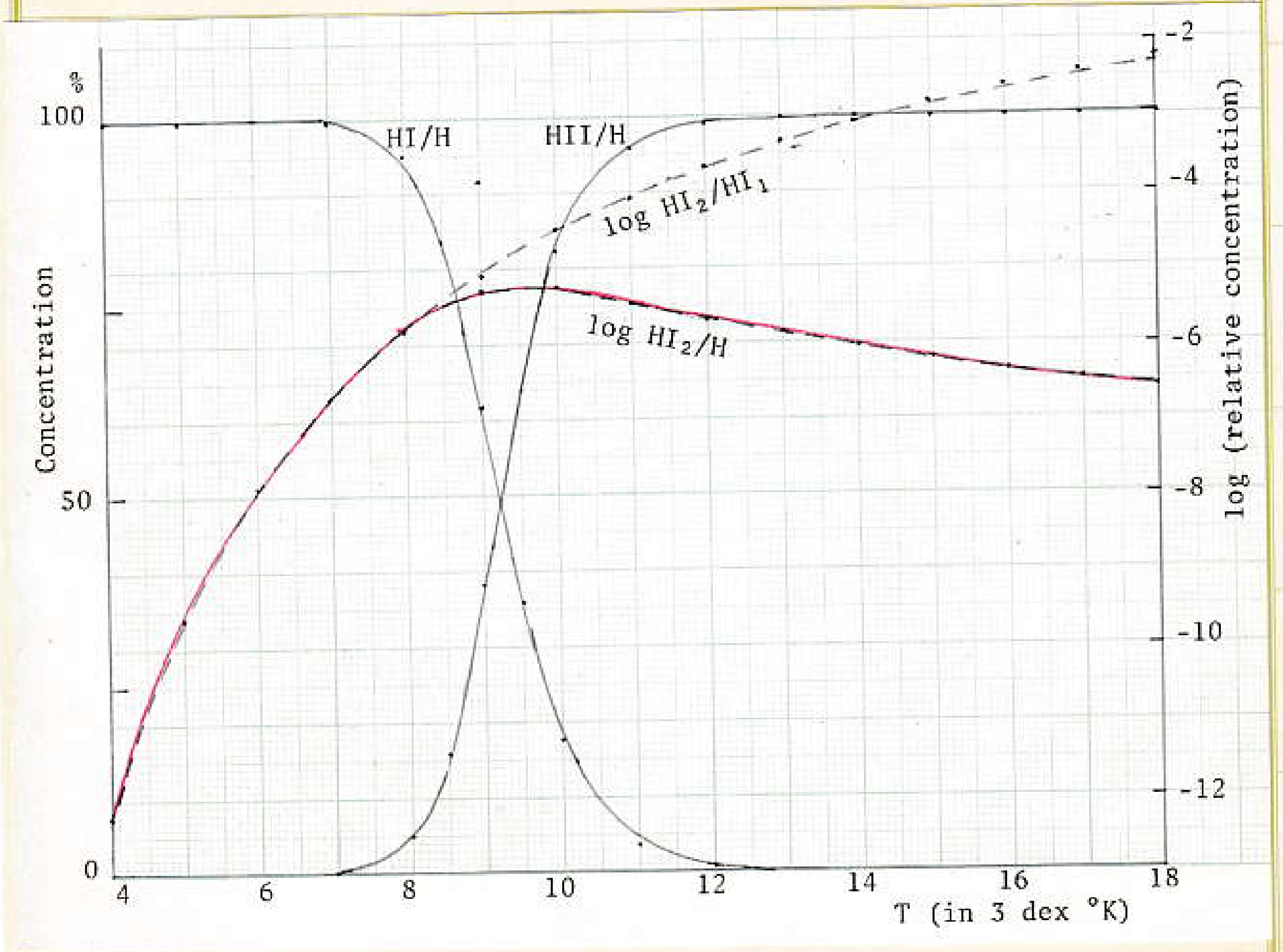
What fraction of H, He, Mg, Si, Fe, Na, K and Ca atoms are neutral and in the singly ionized condition in the atmospheres of I-star (of $T = 5{,}700$ and $p_e = 30$ dynes/cm^2) and II-star (of $T = 10{,}000$ and $p_e = 300$ dynes/cm^2) ?

Element	go	X_o	g1	X_1	g2	X_2	g3	X_3
H	2	13.59	1					
He	1	24.58	2	50.40	1			
Mg	1	7.64	2	15.03	1	80.12	6	109.29
Si	9	8.15	6	16.34	1	33.46	2	45.13
Fe	25	7.90	30	16.18	25	30.64		57
Na	2	5.14	1	47.29	6	71.65		98.88
K	2	4.34	1	31.81	6	46	9	60.90
Ca	1	6.11	2	11.87	1	51.21	6	67

2nd Star	T=10,000	P_e=300				
Element	X_o g_o	X_1 g_1	X_2 g_2	X_3 g_3		
H	13.59, 2	1				
He	24.58, 1	50.40, 2	1			
Mg	7.64, 1	15.03, 2	80.12, 1	109.29,6		
Si	8.15, 9	16.34, 6	33.46, 1	45.13,2		
Fe	7.90,25	16.18,30	30.64,25	57		
Na	5.14, 2	47.29, 1	71.65, 6	98.88,		
K	4.34 2	31.81 1	46 6	60.90,9		
Ca	6.11 1	11.87 2	51.21 1	67 6		
	log	N_1/N_o	log	N_2/N_1	log	N_3/N_2
H	-4.5839	2.61(-5)				
He						
Mg						
Si						
Fe	3.4415	2.76 (3)	- 0.8900	1.29(-1)		
Na						
K						
Ca						

Example (Saha + Boltzmann)

Calculate the fraction of hydrogen atoms excited in the 2nd level of HI, which are capable of absorbing the Balmer lines. Assume P_e = 100 dyn/cm² and T_e = 4000°K to 18000°K (with 1000°K-interval).

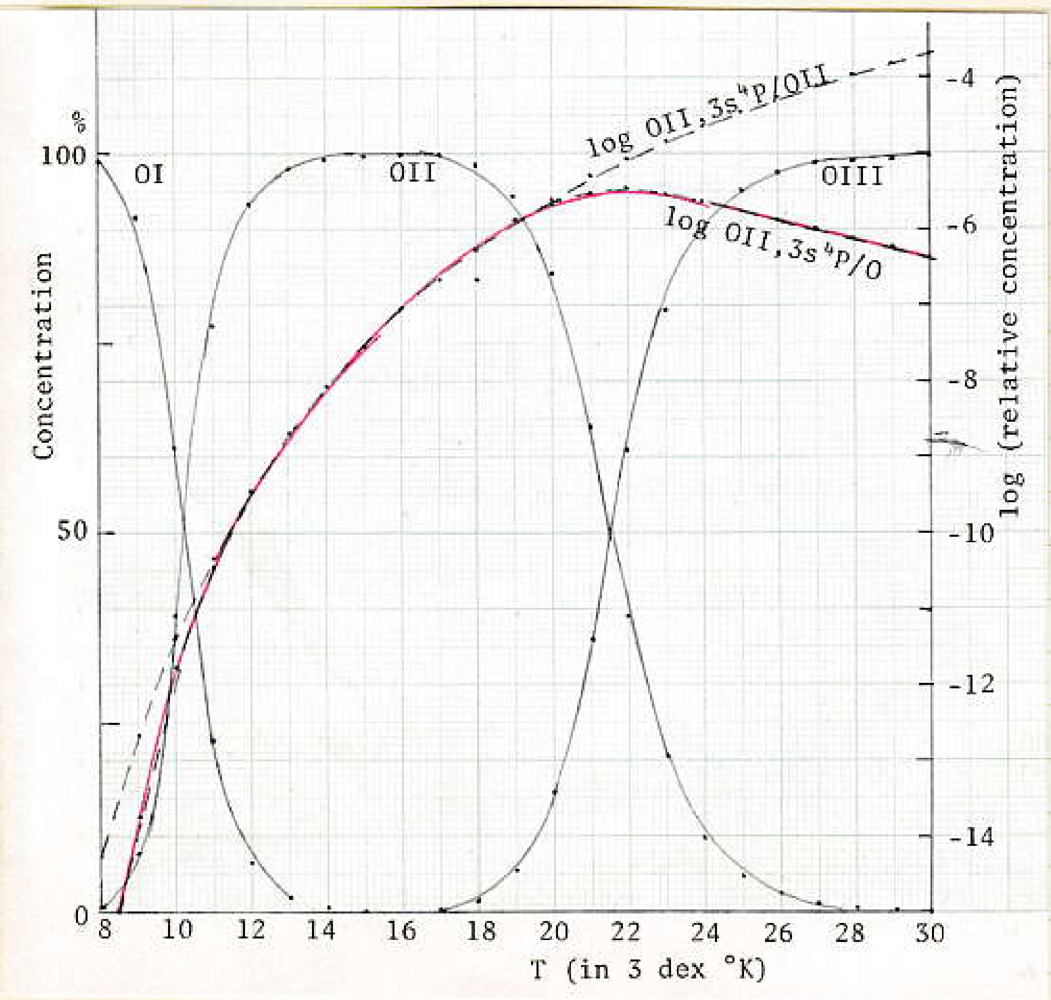


Example (Saha + Boltzmann)

Calculate the fraction of oxygen atoms capable of absorbing 4649 Å, 3s $^4P_{5/2}$ - 3p $^4D_{7/2}$, of OII. The excitation potential of the lower level is 22.90 ev. The ground term of OII is $^4S_{3/2}$. Assume T_e = 8,000°K to 30,000°K with 500°K-interval and P_e = 630 dyn/cm^2.

From A.Q.3. we get:

O II/O I :	χ_r = 13.618 ev	g_{r+1} = 4	g_r = 9
O III/O II :	35.117	9	4
O IV/O III:	54.934	6	9
O II,3s^{4}P/O II	ε = 22.90	6	4



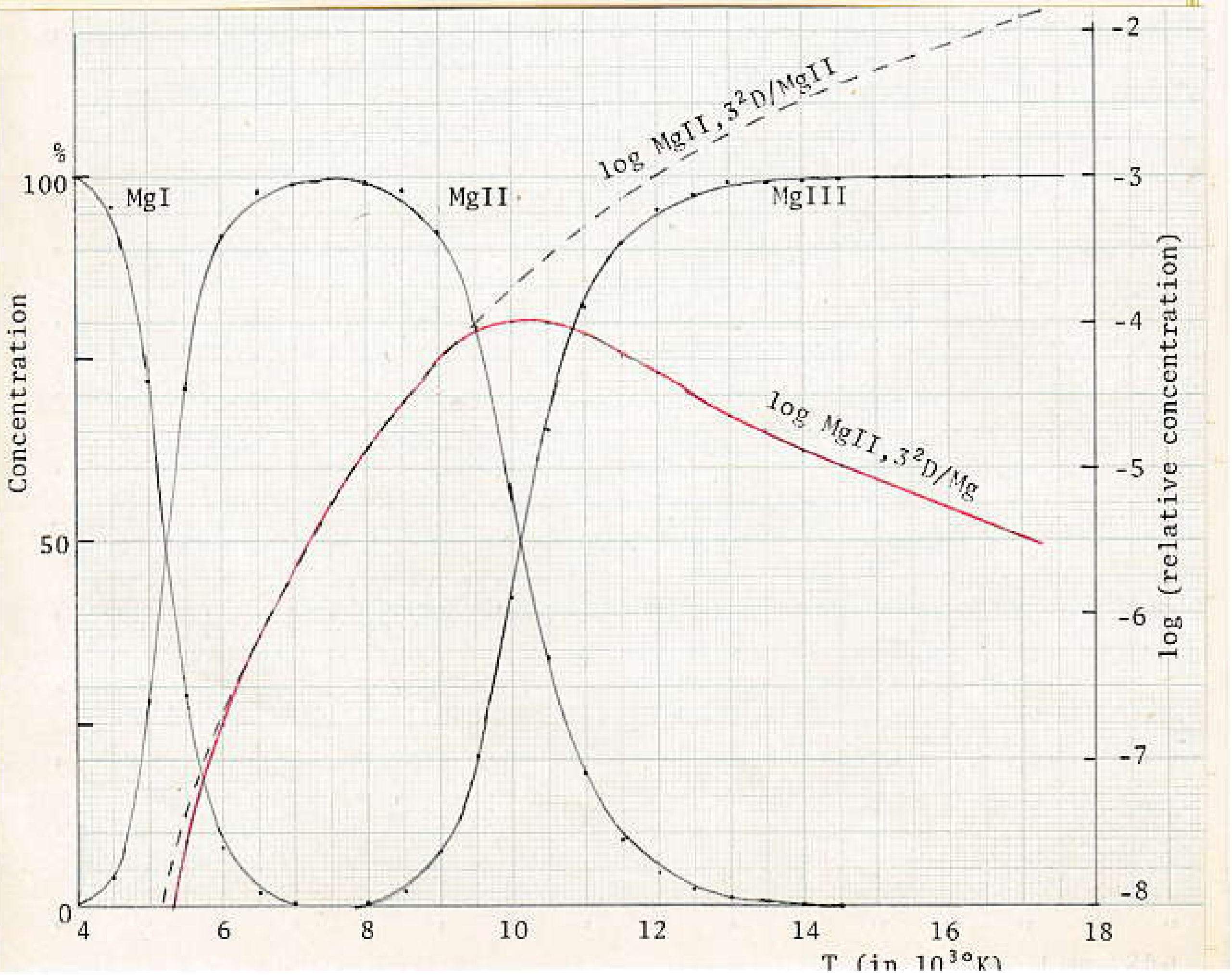
Te	II/I	III/II	I/T	II/T	III/T	logIIa/II	logIIa/T
8000	7.096-3	1.026-15	9.930-1	7.046-3	7.227-18	-14.251	-16.403
9000	8.551-2	3.954-13	9.212-1	7.877-2	3.114-14	-12.648	-13.752
10000	6.442-1	4.764-11	6.082-1	3.918-1	1.867-11	-11.366	-11.773
11000	3.436	2.455-9	2.254-1	7.746-1	1.901-9	-10.316	-10.427
12000	1.416+1	6.683-8	6.597-2	9.340-1	6.243-8	-9.442	-9.472
13000	4.764+1	1.114-6	2.056-2	9.794-1	1.091-6	-8.702	-8.711
14000	1.365+2	1.259-5	7.275-3	9.927-1	1.250-5	-8.068	-8.071
15000	3.443+2	1.042-4	2.895-3	9.970-1	1.039-4	-7.518	-7.519
16000	7.816+2	6.683-4	1.277-3	9.981-1	6.670-4	-7.037	-7.038
17000	1.626+3	3.483-3	6.127-4	9.959-1	3.469-3	-6.613	-6.615
18000	3.141+3	1.521-2	3.136-4	9.847-1	1.497-2	-6.236	-6.243
19000	5.715+3	5.728-2	1.655-4	9.457-1	5.417-2	-5.898	-5.922
20000	9.840+3	1.905-1	8.535-5	8.399-1	1.600-1	-5.595	-5.671
21000	1.622+4	5.675-1	3.933-5	6.379-1	3.620-1	-5.320	-5.515
22000	2.564+4	1.542	1.534-5	3.934-1	6.066-1	-5.070	-5.475
23000	3.917+4	3.855	5.258-6	2.060-1	7.940-1	-4.842	-5.528
24000	5.794+4	8.974	1.730-6	1.003-1	8.997-1	-4.633	-5.632
25000	8.356+4	1.959+1	5.813-7	4.857-2	9.514-1	-4.441	-5.755
26000	1.175+5	4.046+1	2.053-7	2.412-2	9.759-1	-4.263	-5.881
27000	1.618+5	7.943+1	7.684-8	1.243-2	9.876-1	-4.099	-6.004
28000	2.183+5	1.493+2	3.049-8	6.654-3	9.933-1	-3.946	-6.123
29000	2.897+5	2.692+2	1.278-8	3.702-3	9.963-1	-3.804	-6.236
30000	3.776+5	4.677+2	5.650-9	2.133-3	9.979-1	-3.671	-6.342

Example (Saha + Boltzmann)

The strong MgII doublet at 4481 Å arises from the transition $3^2D_{3/2,\ 5/2}$ term to 4^2F term. The excitation potential of the lower term of the transition is 8.83 ev, and the ground term of MgII is $3^2S_{1/2}$. Calculate the fraction of Mg atoms capable of absorbing the line for P_e = 120 dyn/cm² and T_e = 4,000°K to 17,000°K with 500°K-interval.

From A.Q.3 we get:

Mg II /Mg I	:	χ_r = 7.646 ev	g_{r+1} = 2	g_r = 1
MgIII /Mg II	:	15.035	1	2
Mg IV /MgIII	:	80.143	6	1
MgII,3^2D/MgII	:	ε = 8.83	10	2



T	II/I	III/II	I/T	II/T	III/T	IIa/II	IIa/T
4000	2.611-3	3.196-13	9.977-1	2.605-3	8.326-16	3.743-11	9.748-14
4500	4.123-2	5.463-11	9.605-1	3.960-2	2.163-12	6.447-10	2.553-11
5000	3.854-1	3.434-9	7.218-1	2.782-1	9.553-10	6.285-9	1.749-9
5500	2.455	1.040-7	2.895-1	7.106-1	7.390-8	4.050-8	2.878-8
6000	1.171+1	1.818-6	7.868-2	9.213-1	1.675-6	1.913-7	1.763-7
6500	4.460+1	2.080-5	2.193-2	9.781-1	2.034-5	7.117-7	6.961-7
7000	1.423+2	1.703-4	6.978-3	9.929-1	1.691-4	2.195-6	2.179-6
7500	3.937+2	1.066-3	2.531-3	9.964-1	1.062-3	5.824-6	5.803-6
8000	9.692+2	5.362-3	1.025-3	9.936-1	5.328-3	1.368-5	1.359-5
8500	2.166+3	2.251-2	4.513-4	9.775-1	2.200-2	2.906-5	2.841-5
9000	4.462+3	8.121-2	2.072-4	9.247-1	7.509-2	5.678-5	5.250-5
9500	8.582+3	2.579-1	9.262-5	7.949-1	2.050-1	1.034-4	8.217-5
10000	1.556+4	7.345-1	3.705-5	5.765-1	4.234-1	1.773-4	1.022-4
10500	2.683+4	1.095	1.283-5	3.443-1	6.559-1	2.888-4	9.942-5
11000	4.425+4	4.553	4.070-6	1.801-1	8.200-1	4.500-4	8.103-5
11500	7.022+4	1.014+1	1.278-6	8.975-2	9.101-1	6.747-4	6.056-5
12000	1.077+5	2.123+1	4.177-7	4.499-2	9.551-1	9.781-4	4.401-5
12500	1.604+5	4.205+1	1.448-7	2.323-2	9.768-1	1.376-3	3.197-5
13000	2.324+5	7.935+1	5.357-8	1.245-2	9.879-1	1.886-3	2.348-5
13500	3.289+5	1.433+2	2.106-8	6.928-3	9.928-1	2.526-3	1.750-5
14000	4.554+5	2.491+2	8.781-9	3.999-3	9.962-1	3.313-3	1.325-5
14500	6.187+5	4.179+2	3.858-9	2.387-3	9.975-1	4.264-3	1.018-5
15000	8.258+5	6.793+2	1.780-9	1.470-3	9.986-1	5.396-3	7.932-6
15500	1.085+6	1.073+3	8.581-10	9.310-4	9.990-1	6.727-3	6.262-6
16000	1.404+6	1.652+3	4.310-10	6.051-4	9.996-1	8.270-3	5.005-6
16500	1.794+6	2.482+3	2.245-10	4.027-4	1.000	1.004-2	4.044-6
17000	2.265+6	3.650+3	1.209-10	2.739-4	9.997-1	1.205-2	3.301-6